Wall Effect for the Fall of Single Drops

J. R. STROM and R. C. KINTNER, Illinois Institute of Technology, Chicago, Illinois

Measurements were made of the rate of fall of drops of five organic liquids through an aqueous phase contained in eight vertical cylinders of various diameters. Newton's equation for the wall proximity effect for rigid spheres or cylinders predicts values somewhat in excess of the observed. A correction factor equation with the more convenient equivalent spherical diameter is presented. Its use is limited to d/D ratios less than one half. The ultimate velocity of a drop of specific size in an infinite medium can be calculated from that measured in a small tube by multiplying the latter by the ratio of the tube cross-sectional area to the area of the annular space between tube wall and drop.

The importance in chemical engineering science of phenomena connected with the motion of single liquid drops in a liquid field has been stated by many recent authors. Much of the published data on the rise or fall of such drops through liquid media have been taken in vertical cylindrical tubes of small diameter. The relation of such velocities to those in an infinite medium or in tubes of other sizes is an uncertain matter. If the field liquid be other than water, the expense of filling large tanks is excessive. If the liquid be also flammable, the safety hazard may be great. In order that ultimate velocities measured in tubes of small diameter may be interpreted in terms of those in an infinite medium and to serve as a guide in the design of experiments, the effect of wall proximity on the fall of liquid drops through water was determined. Very small drops have not been included in this work since they can be investigated in reasonably small tubes without incurring an excessive wall effect. The interconnected factors of oscillation, deformation, internal circulation, skin friction, pressure drag, and high inertia forces were present in nearly all runs.

Several recent papers (*2–5*) have included reviews of the effects of the above variables and no discussion of them seems necessary here.

In a previous investigation (*7*) a summary was given of the work of various authors on the reduction of the terminal velocity of rigid spheres due to a near-by cylindrical boundary. Only the equation of Newton (*6*)

$$\frac{U}{U_\infty} = \frac{1}{K} = \left[1 - \left(\frac{d_F}{D}\right)^2\right]\left[1 - \frac{1}{2}\left(\frac{d_F}{D}\right)^2\right]^{\frac{1}{2}} \quad (1)$$

was derived for conditions approaching those of the current work. The empirical equation of Uno and Kintner (*7*) for the calculation of the wall effect for the rate of rise of single air bubbles through liquids seems to be less useful for liquid-liquid systems. Gas bubbles exhibit no appreciable wall effect for d/D values less than about 0.1 and liquid-liquid systems do not show a variation in wall effect with variation in tube diameter and interfacial tension as reported for gas bubbles.

The projected area of the deformed and oscillating drops is close to circular and the diameter of the sphere or cylinder in Newton's equation should be the diameter of this projected frontal area. The eccentricity of drops moving in a liquid medium was measured by Keith and Hixson (*3*) and by Klee and Treybal (*4*). The latter presented a correlation which has been used here to convert the easily determined equivalent spherical diameter (d) to the more correct frontal diameter (d_F).

EXPERIMENTS

The terminal velocity measurements were made in cylindrical glass tubes of 1.04, 1.49, 2.09, 3.64, 4.91, 6.90, 9.50, and 15.25 cm. average I.D. Each was 4 ft. long and mounted on a metal base plate for convenience. Terminal velocities of the drops were measured by timing, with a hand-actuated electric stopwatch, the distance of fall between two marks on the tubes. The distance of fall was from 60.25 to 66.25 cm., according to the tube used. The marks were placed about 30 cm. from the top and bottom to allow for end effects (*2*). The electric stopwatch, graduated to 0.01 sec., permitted readings of elapsed time resulting in a maximum variation of +2.48% and an average variation of ±1.03% from the

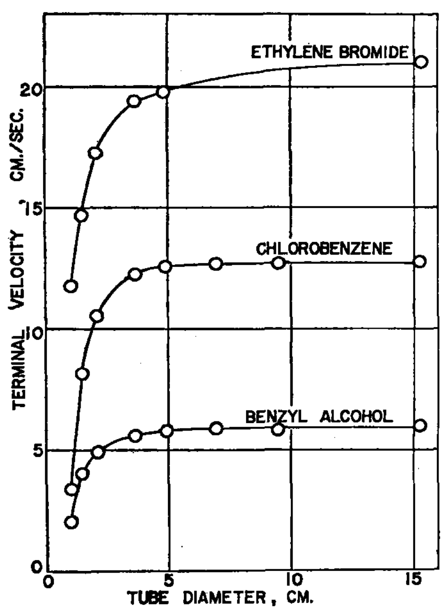

Fig. 1. Terminal velocity vs. tube diameter for a constant drop size (d = 0.749 cm.).

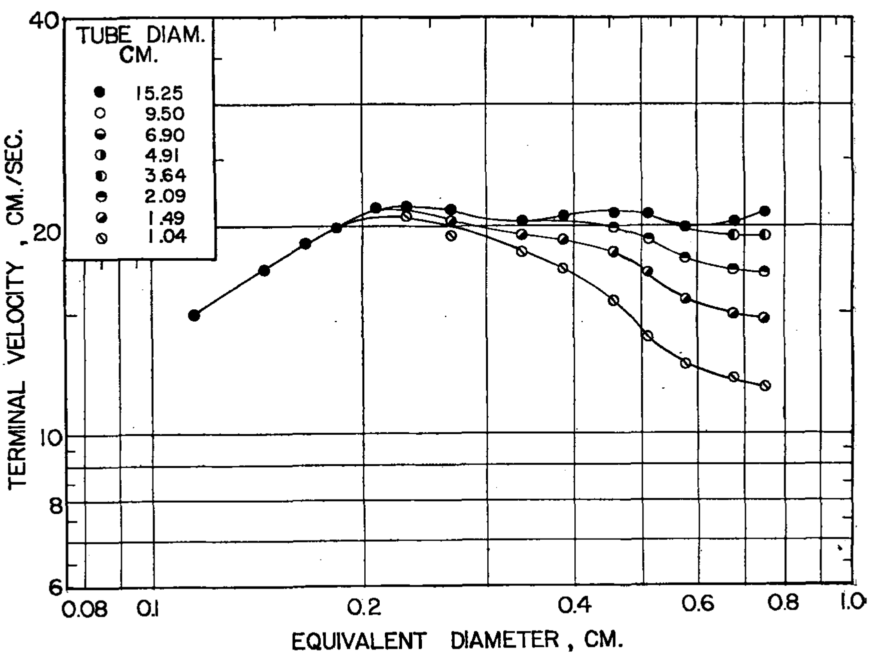

Fig. 2. Terminal velocity of ethylene bromide drop in water.

arithmetic average velocity. Each of the points in Figures 1 to 4 represents the average of twenty observations, except for a few which represent only ten such values. Drop volumes were measured by small, calibrated pipettes and the drops delivered into small glass pouring vessels as described by Hu and Kintner (*2*). The vessels were then carefully placed into the tubes and the drops delivered by the pour technique. Tests were made showing that the terminal velocities of drops delivered by pouring and by nozzles are identical.

The five organic liquids used (ethylene bromide, chlorobenzene, benzyl alcohol, *o*-nitrotoluene and bromobenzene) were of reagent grade. The phases were mutually saturated in all cases. Liquid viscosities were measured with Ostwald-Cannon-Fenske tubes, densities with a pycnometer, and interfacial tensions with a Cenco-Du Nuoy Ring Tensiometer. The physical properties are summarized in Table 1.

DATA

If terminal velocity be plotted as ordinate and tube I.D. as abscissa for a constant drop size, curves like those shown in Figure 1 should be obtained.* Each curve approached U_∞ asymptotically. It also approaches a value of tube diameter that is about equal to the drop frontal diameter as estimated from the correlation of Klee and Treybal (*4*). Consideration of these curves and a comparison with the data of previous authors (*1*, *2*, , *4a*, *5*) shows that the terminal velocities in the 15.25 cm. (6-in.) tube may be treated as if it were of infinite extent. For chlorobenzene drops, Licht and Narasimhamurty (*5*) obtained velocities about 10% higher than those reported here.

Curves of terminal velocity vs. drop equivalent diameter with tube I.D. as the third variable are shown in Figures 2, 3, and 4. The *o*-nitrotoluene-water and bromobenzene-water systems were used for checking purposes and only a few points taken. The curve of U_∞ (15.25 cm. tube) shows the normal shape for drops. Velocity increases with *d* for small drops. The curve continues to rise until a peak is reached after which an increase in drop size results in a small decrease in velocity. Terminal velocity is nearly constant and independent of drop size in the highest ranges of the latter.

For both the chlorobenzene-water and benzyl alcohol-water systems, the wall effect in the larger tube sizes results in an increased velocity reduction as drop size is increased. The velocity curves have the same general shape as the U_∞ curve. In small tubes, a point is reached at which an increase in drop size results in a much larger decrease in velocity and the curve is bent sharply downward. Since the drop is not spherical in shape, the eccentricity of the drop causes the effective frontal diameter (d_F) to be larger than the equivalent spherical diameter (d) and the curve of U vs. d is lowered before reaching a value of d equal to D, as shown in Figures 3 and 4. This point was never reached in the ethylene bromide-water system as it was found to be impossible to deliver drops larger than 0.75 cm. diam. without

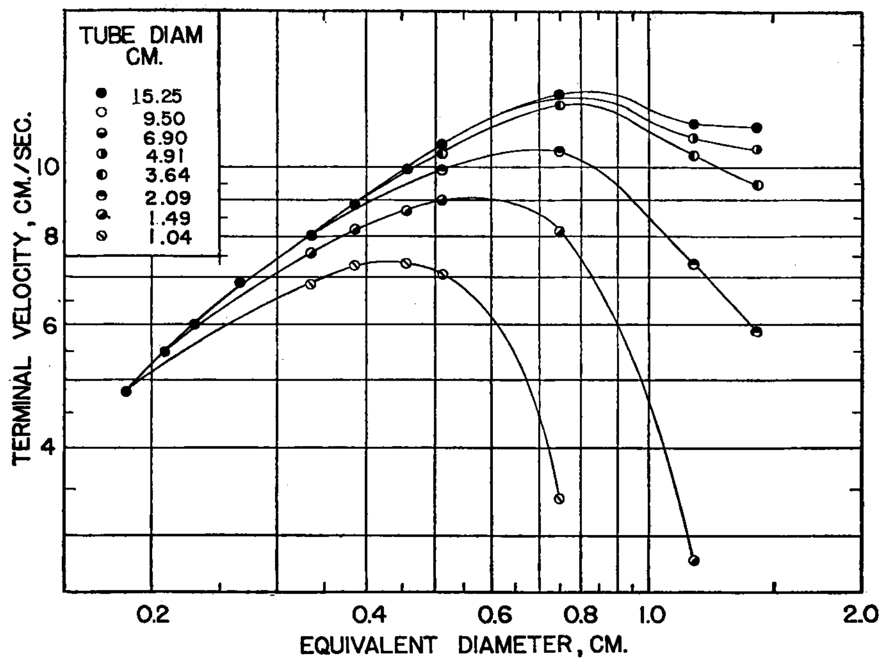

Fig. 3. Terminal velocity of chlorobenzene drop in water.

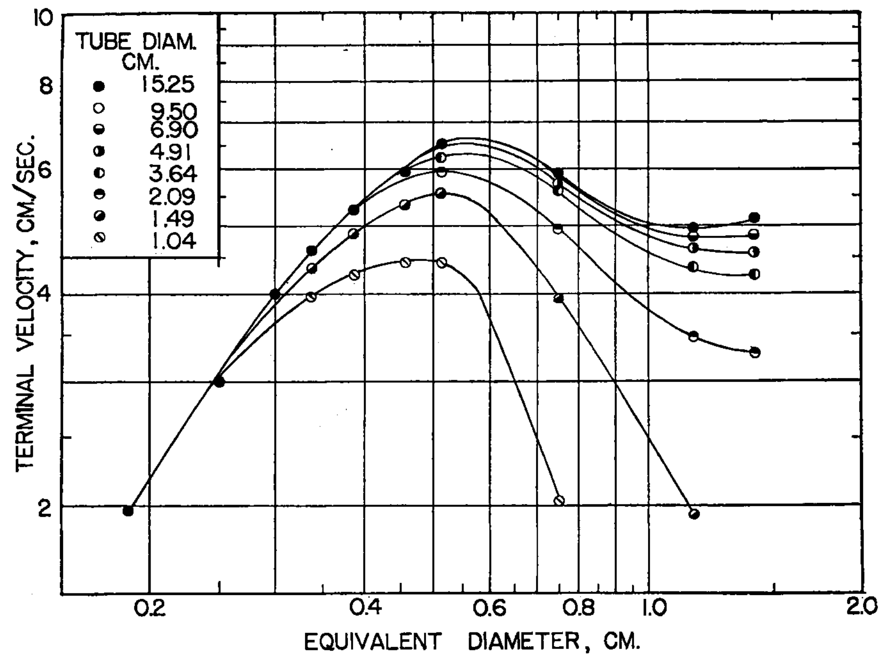

Fig. 4. Terminal velocity of benzyl alcohol drop in water.

TABLE 1. PHYSICAL PROPERTIES

(in C.G.S. units)

No.	System	ρ_0	ρ	$\mu_0 \times 10^2$	$\mu \times 10^2$	σ_i	T
1	Ethylene bromide	2.1611	0.9972	1.570	0.858	30.4	26.8
2	Chlorobenzene	1.0981	0.9965	0.748	0.864	31.1	26.3
3	Benzyl alcohol	1.0398	0.9990	3.620	0.912	5.8	26.8
4	*o*-Nitrotoluene	1.1576	0.9970	2.036	0.900	26.5	24.7
5	Bromobenzene	1.4881	0.9971	1.072	0.896	37.9	25.0

*Tables of the original data giving values of D, d, U, d/D, $[1 - (d/D)^2]$, U/U_∞ may be obtained as document 5597 from the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., for $1.25 for 35 mm. microfilm or photoprints.

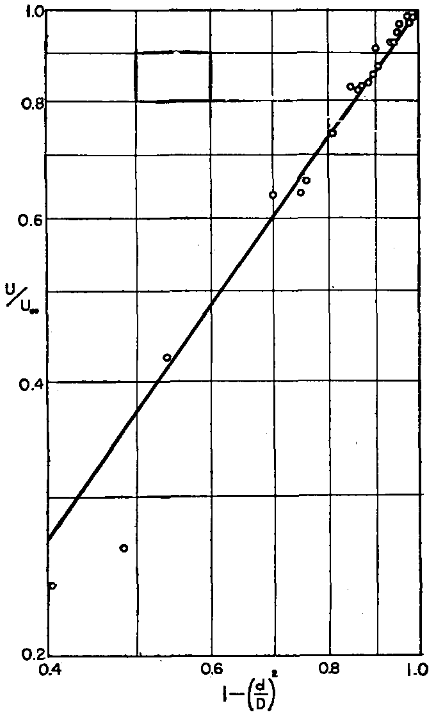

Fig. 5. Wall effect in chlorobenzene-water system.

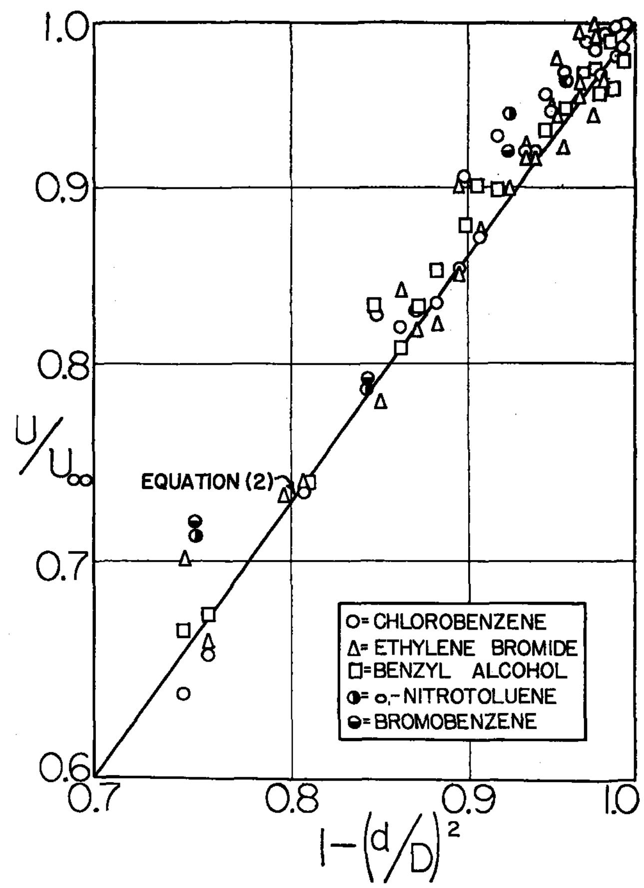

Fig. 6. Correlation based on equivalent diameter.

breaking them. The curves for this system are shown in Figure 2.

CORRELATION

Since the reduction in velocity is caused by a reduction in the area available for fluid to flow around the drop, the method of correlation of results is based on the area of the annular space with reference to the drop diameter and tube diameter. This is particularly convenient because a plot of Equation (1) on the suggested coordinates ($U/U_\infty = 1/K$ as ordinate vs. $(D^2 - d_F^2)/D^2 = 1 - (d_F/D)^2$ as abscissa) is a straight line for values of $(d_F/D)^2$ less than three tenths. As indicated below, drops of such a size that (d/D) is greater than one half (equivalent to $(d/D)^2$ greater than one fourth) are affected by such shear forces that they are subject to division. The chlorobenzene-water system, with a high interfacial tension and low density difference, did not exhibit such a tendency and was used as the basis for locating the line in Figure 5. Only in such a system could the terminal velocities of large single drops in small tubes be measured without involving such a large ratio of inertial forces to surface forces as to preclude the attainment of verifiable results. The line of Figure 5 was located by the method of selected points. If d/D be zero, $U = U_\infty$ and K is unity. A straight line passing through this point was visually located so that the algebraic sum of the deviations was nearly zero. The average deviation of the points from this line was 2.57%.

The experimental data for all systems, plotted in Figure 6, resulted in a band of points which can be represented by the same line as the chlorobenzene-water data. The equation of the line was found to be

$$\frac{U}{U_\infty} = \frac{1}{K} = \left[1 - \left(\frac{d}{D}\right)^2\right]^{1.43} \qquad (2)$$

The average deviation of the seventy-eight points in Figure 6 from this equation was 1.91%.

If the ratio (d/D) be replaced by (d_F/D) and the exponent be changed from 1.43 to 1.24, the resulting equation can be used to calculate values of K which agree with those of Newton's equation for values of (d/D) less than one half. This limit corresponds to values of $[1 - (d/D)^2]$ of 0.75, below which the drops should not be considered as falling freely.

If frontal diameters of the generally ellipsoidal drops be estimated by the correlation of Klee and Treybal (*4*) and used in Equation (1), a better evaluation of the applicability of Newton's equation may be had. This is shown in Figure 7, from which it is evident that the velocity reduction is less than one might expect for a rigid sphere of the same frontal diameter. The average deviation of the points from Newton's equation was 3.14%.

The points in Figure 7 can also be represented quite well by the equation

$$\frac{U}{U_\infty} = \frac{D^2 - d_F^2}{D^2} \qquad (3)$$

although not as well as by Equation (2) which is based on equivalent spherical diameter. The right member of Equation (3) is the ratio of the area of the annular space between drop and wall to the tube cross-section. Thus the velocity of a drop of specific size in an infinite medium can be calculated by multiplying the velocity measured in a small tube by the ratio tube cross-sectional area to the area of the annulus. Equation (3) also serves to relate the drop velocity in tubes of various sizes. The average variation of the points from this equation was 2.14%.

Unpublished data obtained by previous experimenters in these laboratories also fall within the band of points of Figures 6 and 7. Braida (*1*) has reported measurements of terminal velocities of drops of "Mixture I" (carbon tetrachloride and tetrabromoethane), *o*-nitrotoluene, and carbon tetrachloride falling through water in tubes of 2.4, 4.1, 5.9, and 7.6 cm. I.D. The highest ratio of d/D for drops of Mixture I was 0.0963 and the wall effect was less than 1%. The data for carbon tetrachloride drops fell closely along the line of Figure 6. The points for *o*-nitrotoluene drops were widely scattered and no conclusion could be drawn from them

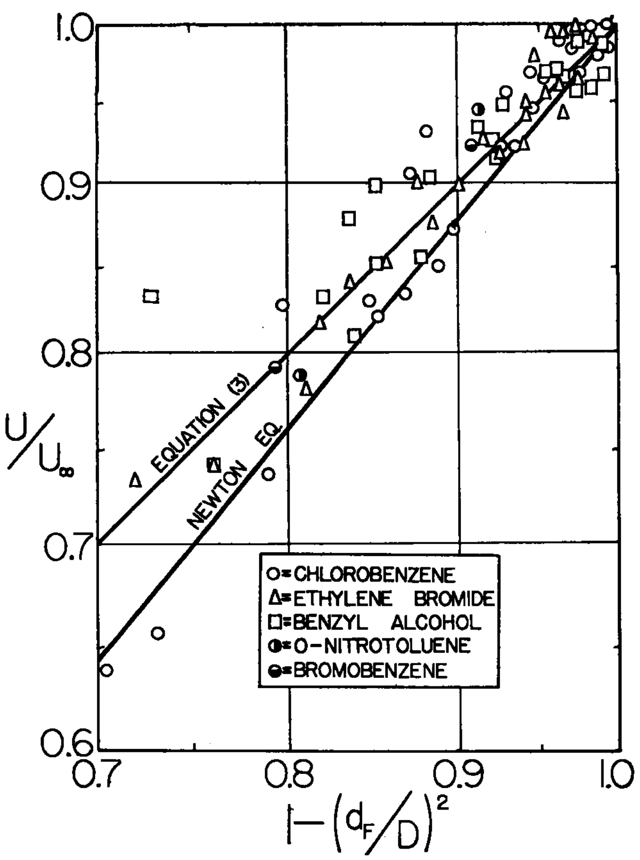

Fig. 7. Comparison with Newton's equation.

DISCUSSION

The motion of a large liquid drop through a field liquid inside a cylindrical boundary calls into play all the factors of deformation, oscillation, internal circulation, acceleration and deceleration of the field fluid, pressure drag, skin friction, and the physical properties of the liquids under study. As the drop moves in the cylinder, the continuous phase must flow around the drop in the annular space. The field fluid has zero initial velocity and, as the drop passes, it is accelerated to a maximum velocity and then decelerated to zero. As the drop passes a point in the tube, a volume of the continuous phase equal to the volume of the drop will be elevated a distance equal to the drop volume divided by the cross-sectional area of the tube. If the drop be large enough or the cylinder small enough, turbulences may be of sufficient magnitude to cause the drop to break up. Such an effect was noted in the course of the experiments. The turbulence due to the rush of field fluid around the drop was sufficient to overcome surface forces and shearing of the drop occurred. This situation prevails in small cylinders for relatively large drops with high densities or low interfacial tensions.

The terminal velocity of a liquid drop in a vertical tube is also a function of the mode of descent of the drop. A drop of specified size may not have the same type of motion in cylinders of differing diameters. If the drop be small enough, it will be spherical in shape and its velocity in the absence of a wall effect will be that of an equivalent rigid sphere, as may be shown by a plot of drag coefficient vs. Reynolds number. If the drop be somewhat larger, deformation takes place and the drop is no longer spherical but resembles an oblate ellipsoid with the minor axis oriented in a generally vertical direction. A small drop of low density shows little effect in mode of descent as the boundary is brought nearer. But if the annular space between drop and cylinder wall is small enough, the drop will start to oscillate. A drop of higher density (or $\Delta\rho$) will undergo this occurrence before the d/D ratio is as large as in systems of low $\Delta\rho$. Systems of low interfacial tension also exhibit this effect at low d/D values. The larger drops also show a marked difference in mode of descent in that the drop pitches and rolls as it falls. Such random motions are believed to be the cause of some of the scatter in the data.

SUMMARY

Careful measurements were made of ultimate velocities of large drops of three organic liquids falling through a continuous aqueous phase contained in vertical cylinders of several diameters. Limited data on drops of two other liquids are reported for checking purposes. Newton's equation (6) for the effect of wall proximity on the velocity of a rigid sphere or cylinder gave velocity reductions in excess of the experimental values. Only frontal diameters (or projected frontal areas) may be properly used in Newton's equation which was derived for conditions of negligible viscous effects.

With the use of the much more convenient equivalent spherical diameter, the wall effect may be more closely expressed by Equation (2) for values of (d/D) less than one half. If values of (d/D) be greater than 0.5, excessive deformation of a type not present in large containers causes any correlation to be of doubtful value.

According to Equation (3), the ultimate velocity in an infinite tank is equal to that in a tube of small size multiplied by the ratio of the tube cross-sectional area to the area of the annular space between drop and tube wall. The use of a calculated drop frontal diameter introduces additional scattering of the points, however, and the use of Equation (2) or of Figure 6 is recommended in all cases.

ACKNOWLEDGMENT

Thanks are due S. J. Macek for his assistance in the gathering of experimental evidence.

NOTATION

d = drop equivalent diameter, i.e., diameter of a sphere equal in volume to the volume of the drop (cm.)
d_F = frontal diameter (major axis) of the ellipsoidal drop (cm.)
D = tube I.D. (cm.)
K = wall correction factor = U_∞/U
U = gross terminal velocity of drop (cm./sec.)
U_∞ = gross terminal velocity of drop in a medium of infinite extent. (cm./sec.)
T = temperature, °C.
$\Delta\rho$ = difference in density between drop and aqueous field phase (g./cc.)
ρ = density
μ = viscosity
σ_i = interfacial tension
$_0$ = organic phase
0 = water phase

LITERATURE CITED

1. Braida, Leno, M.S. thesis, Univ. Toronto (1956).
2. Hu, Shengen, and R. C. Kintner, *A.I.Ch.E. Journal*, **1**, 42 (1955).
3. Keith, F. W., and A. N. Hixson, *Ind. Eng. Chem.*, **47**, 258 (1955).
4. Klee, A. J., and R. E. Treybal, *A.I.Ch.E. Journal*, **2**, 444 (1956).
4a. Korchinski, I.J.O., Ph.D. thesis, Univ. Toronto (1955).
5. Licht, William, and G. S. R. Narasimhamurty, *A.I.Ch.E. Journal*, **1**, 366 (1955).
6. Newton, Isaac, "Mathematica Principia," Univ. Calif. Press, p. 348 (1934).
7. Uno, Seiji, and R. C. Kintner, *A.I.Ch.E. Journal*, **2**, 420 (1956).

Manuscript received July 8, 1957; revision received Dec. 20, 1957; paper accepted Feb. 6, 1958.